By *R*, I underſtand the Radius of the Circle Generant; and by *s*, the Right Sine of the Arch *B A*, whoſe Verſed Sine is *V A*.

And, where ever in my whole Diſcourſe of the *Cycloid*, or the Reſtored Trilinear (which is a Figure of Archs, and a Figure of Verſed Sines) the Arch *a* is no Ingredient in the deſignation; ſuch part or portion of them is capable of being Geometrically ſquared. But when I exclude *a*, I do therein exclude *P* (for that is an Arch alſo) and $f=a+s$, and $e=a-s$, becauſe *a* is therein included.

Mr. *Caſwell*, (not being aware that I had ſquared theſe Figures) had done the ſame by a Method of his own, (which he ſhewed me lately) which I would have inſerted here, but that he thought it not neceſſary; and inſtead thereof, hath given me the Quadrature of a Portion of the *Epicycloid* (which you will receive with this) and, I think, it is purely new.

IV. *The Quadrature of a Portion of the Epicycloid. By Mr.* Caſwel.

SUppoſe *DPV* to be half of an exterior Epicycloid, *VB* its Axis, *V* the Vertex, *VLB* half of the generant Circle, *E* its Center; *DB* the Baſe, *C* its Center: Biſect the Arc of the Semicircle *VB* in *L*, and on the Center *C* through *L* draw a Circle cutting the Epicycloid in *P*: Then I ſay the Curvilinear Triangle *VLP* will be $= BEq \text{ in } \frac{CE}{CB}$; that is, the Square of the Semidiameter of the generant Circle will be to the Curvilinear Triangle *VLP*, as *CB* the Semidiameter of the Baſe, to *CE*: which *CE* in the exterior Epicycloid is the

the Sum of the Semidiameters of the Baſe and Generant, but in the Interior Epicycloid *Dpu*, 'tis the difference of the ſaid Semidiameters.

COROLLARY.

In the Interior Epicycloid, if *CE* is $\frac{1}{2}$ *CB*, the Epicycloid then degenerating into a right Line, the Quadrature of the Triangle *lpu* will be in effect the ſame with the Quadrature of *Hippocrates Chius*.

COROL. II.

If the Semidiameter of the Baſe is ſuppoſed infinite, the Epicycloid then being the common Cycloid, the Area of the ſaid Triangle will be equal to the Square of the Radius of the Generant, and ſo it falls in with that Theorem which *Lalovera* found, and calls *Mirabile*.

Though I do not think the aboveſaid Quadrature can eaſily be deduced from what has been yet publiſhed of the Epicycloid, I have not added the Demonſtration; but think it enough to name a general Propoſition from whence I deduced it, *viz.* The Segments of the Generant Circle are to the Correſpondent Segments of the Epicycloid, as *CB* to 2*CE*+*CB*. For Example, ſuppoſe *FmG* the Poſition of part of the Generant when the point *F* of the Exterior Epicycloid was deſigned, then the Segment *FmGn* is to the Segment *DFnG*:: as *CB* to 2*CE*+*CB*.

And conſequently the whole Epicycloid to the whole Generant in the ſame Proportion: Which is the only caſe demonſtrated by Monſieur *De la Hire*.

It follows alſo that in the Vulgar Cycloid, its Segments are triple of the Correſpondent Sectors of the Generant, which was firſt ſhewn by Dr. *Wallis*.

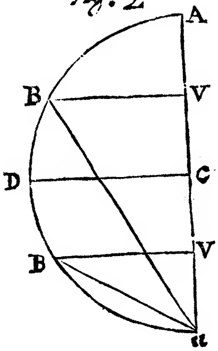
A Demonſtration hereof, with a General Propoſition for all Curves of this kind, ſhall be given in the next Tranſaction. V. A.

[illegible]

[illegible]

Fig. 2

Pag. 88.



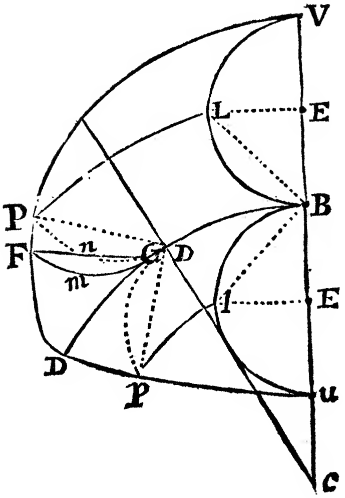


Fig. I

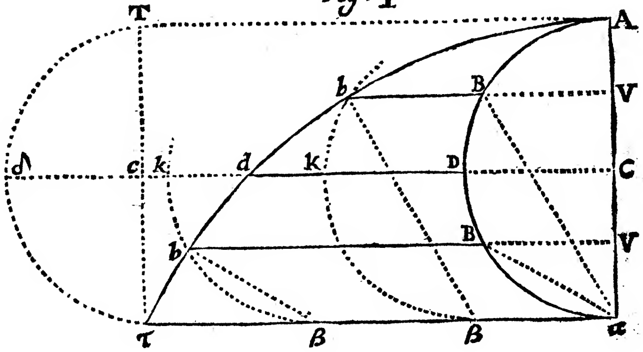


Fig. 3